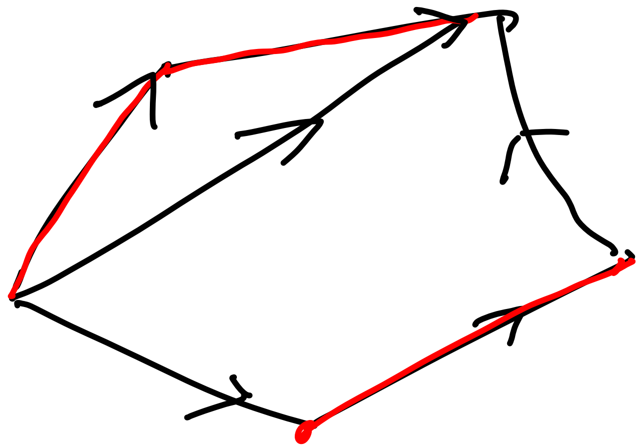
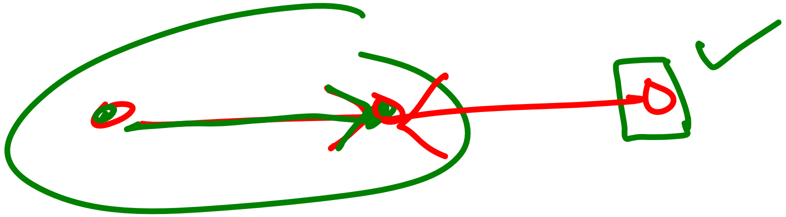


$$|MDPC(G)| \longleftrightarrow MPDC(G)$$

$$\boxed{\underline{MPDC}(G) \le \alpha(G)}$$

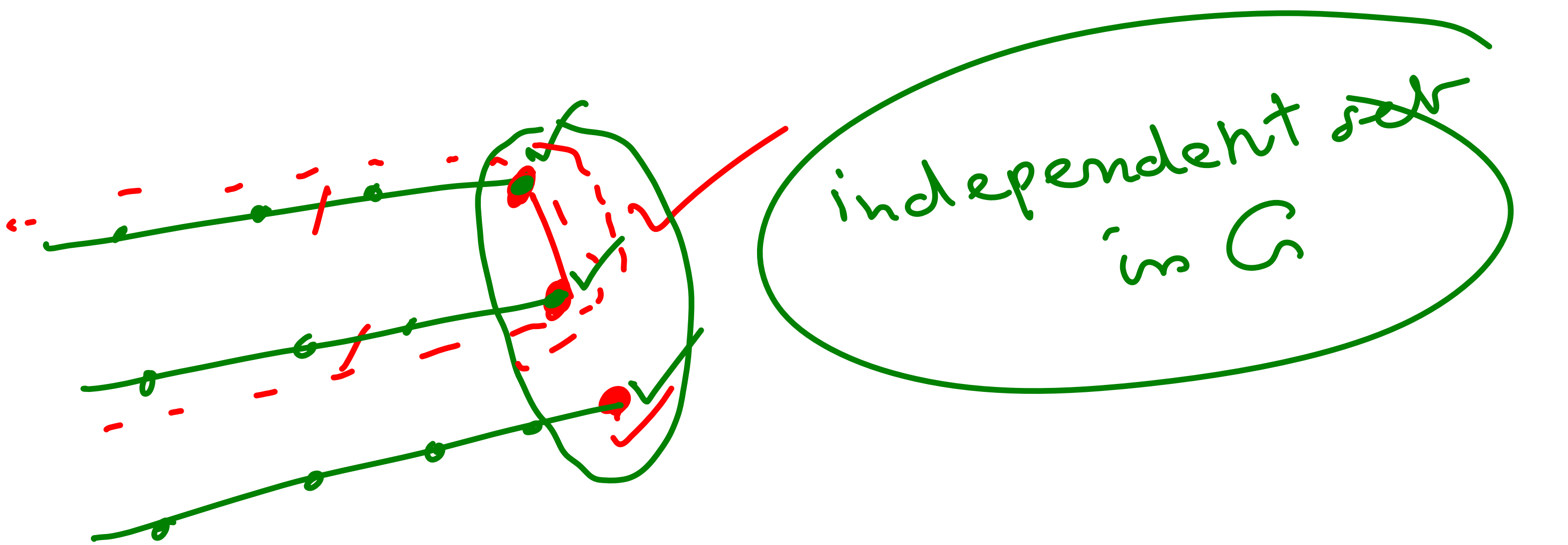


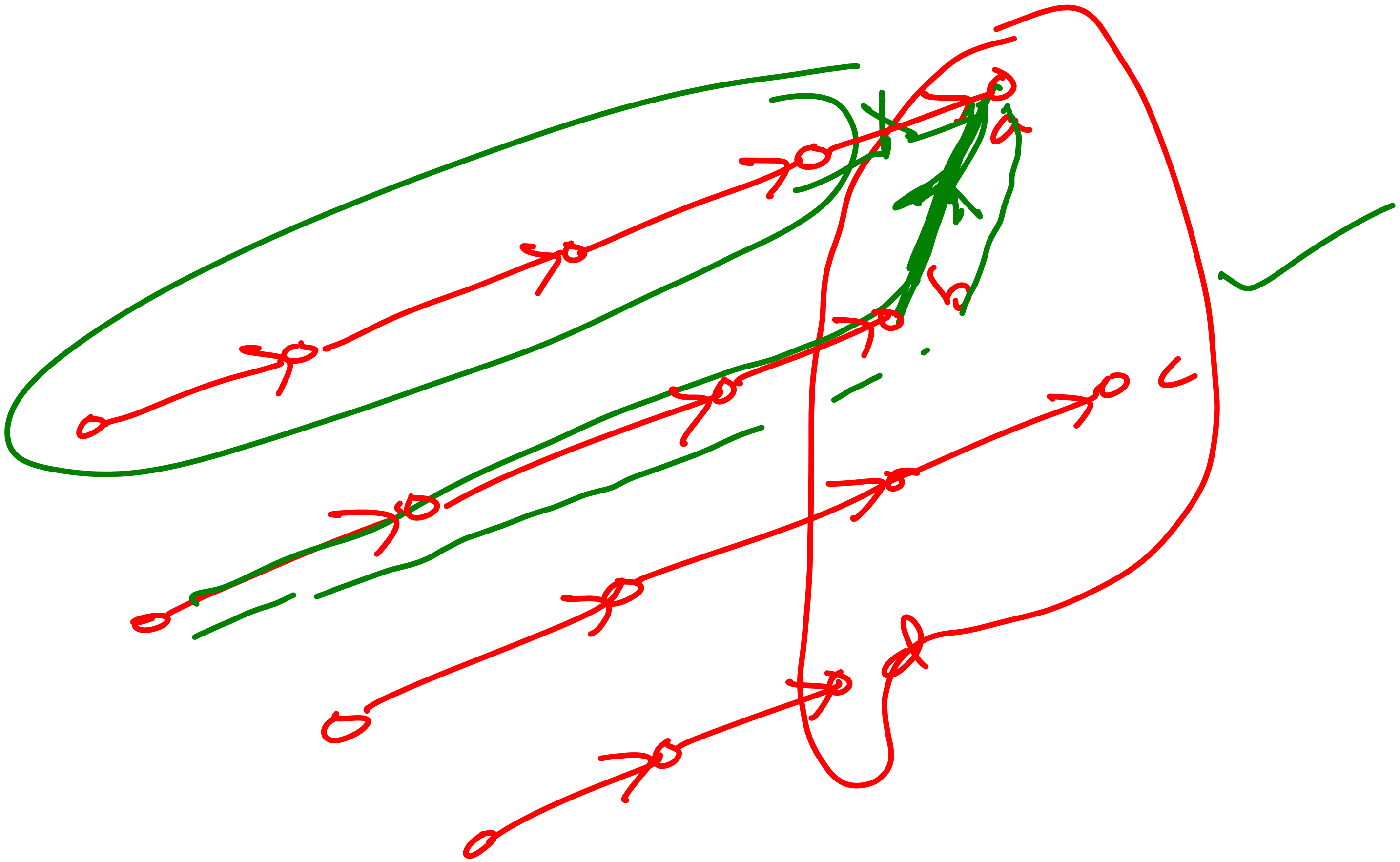
$$MBPC(G) = 2$$

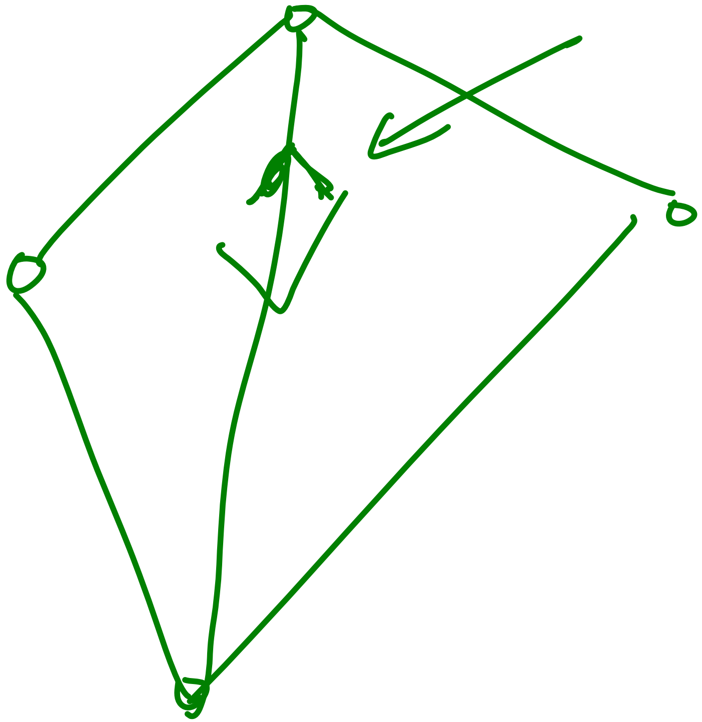
$$MPC(G) = 1$$

G

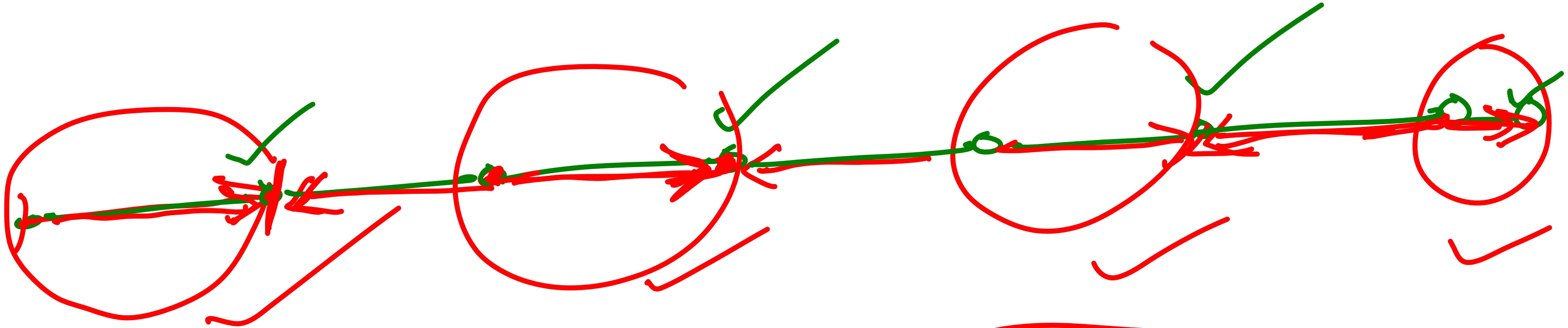
$\mathrm{mpc}(G) \leq \alpha(G)$?


independent set in G





2 ways of orienting each edge



$MPC(G) = 1$

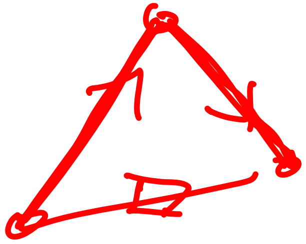
$\alpha(G) = \frac{n}{2}$ ✓

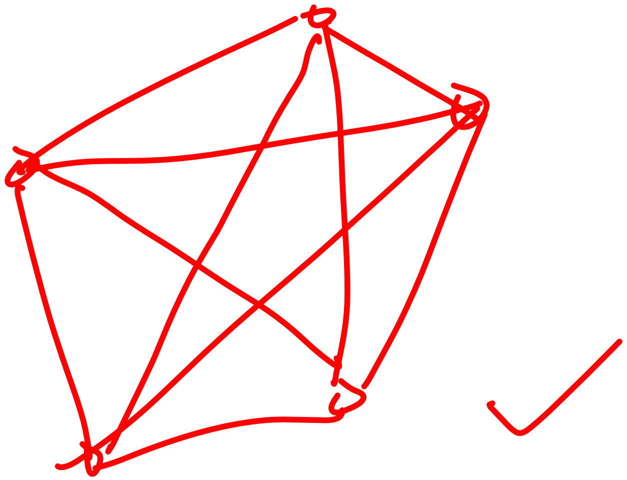
$MDPC(G) = \frac{n}{2} \leq \alpha(G)$

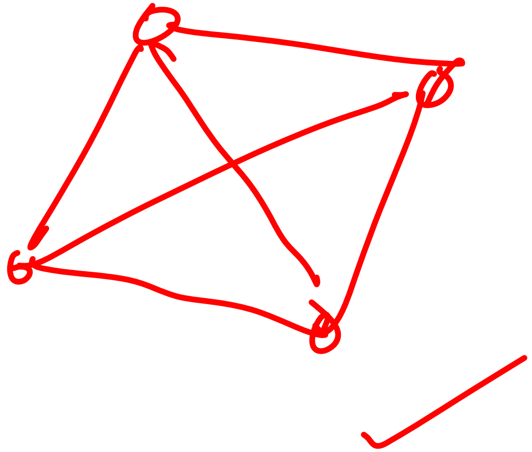
K_n

$\alpha(K_n) = 1$

$\boxed{\text{Hamiltonian path}}$ — which visits each vertex once & only once

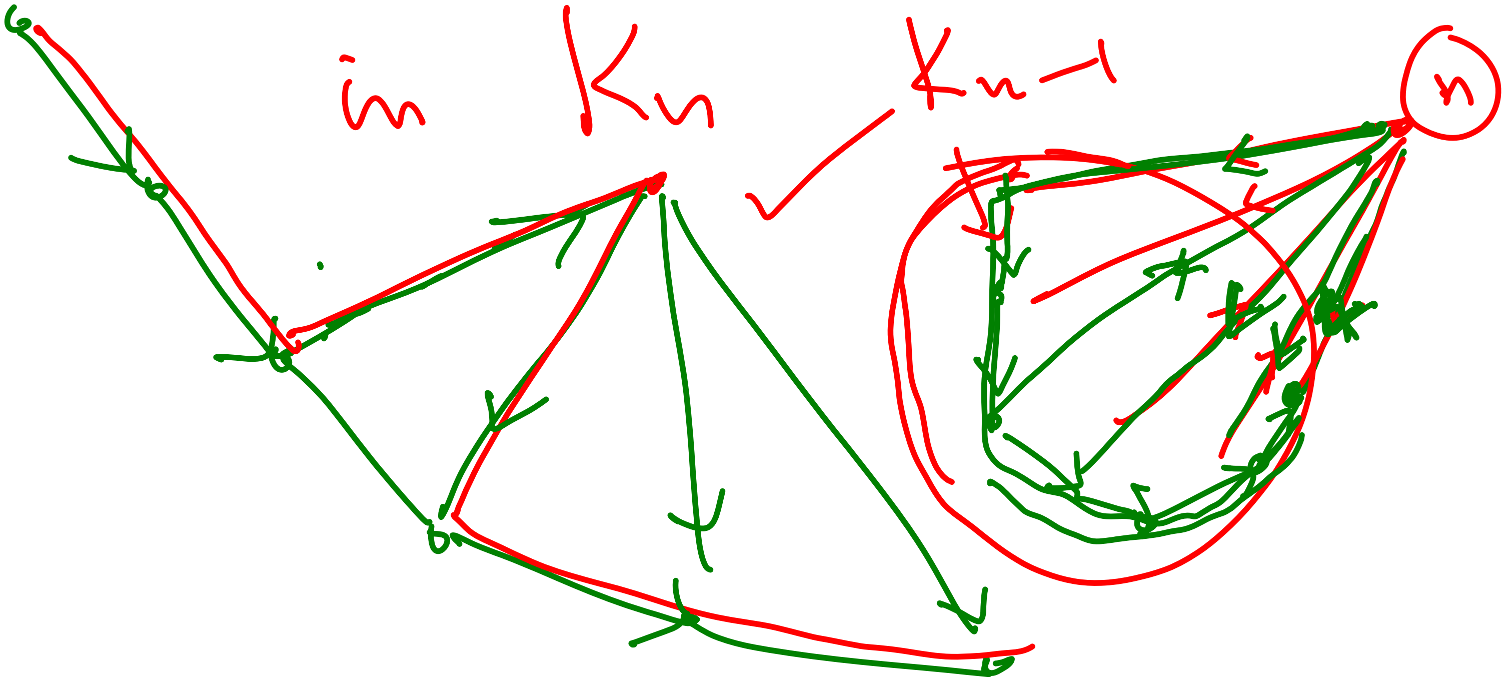


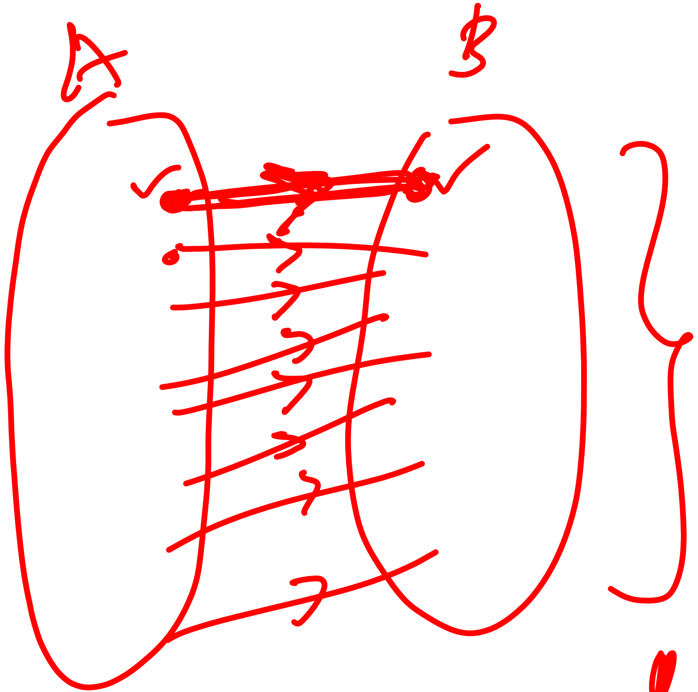




K_{n-1}

in K_n





$\alpha(G)$

$\mathrm{MDPC}(G)$

$= \alpha'(G) + n - 2\alpha'(G)$

$= n - \alpha'(G) = n - \beta(G)$

$\mathcal{P}$ is minimal path cover

then $\exists$ an indepent set S

such that for $P \in \mathcal{P}$,

$S \cap P \neq \emptyset$ $\boxed{\alpha(G) \geq |S| \geq |\mathcal{P}|}$

S

$x \in S$,

$\cancel{S - \{x\}}$

not a vertex cover

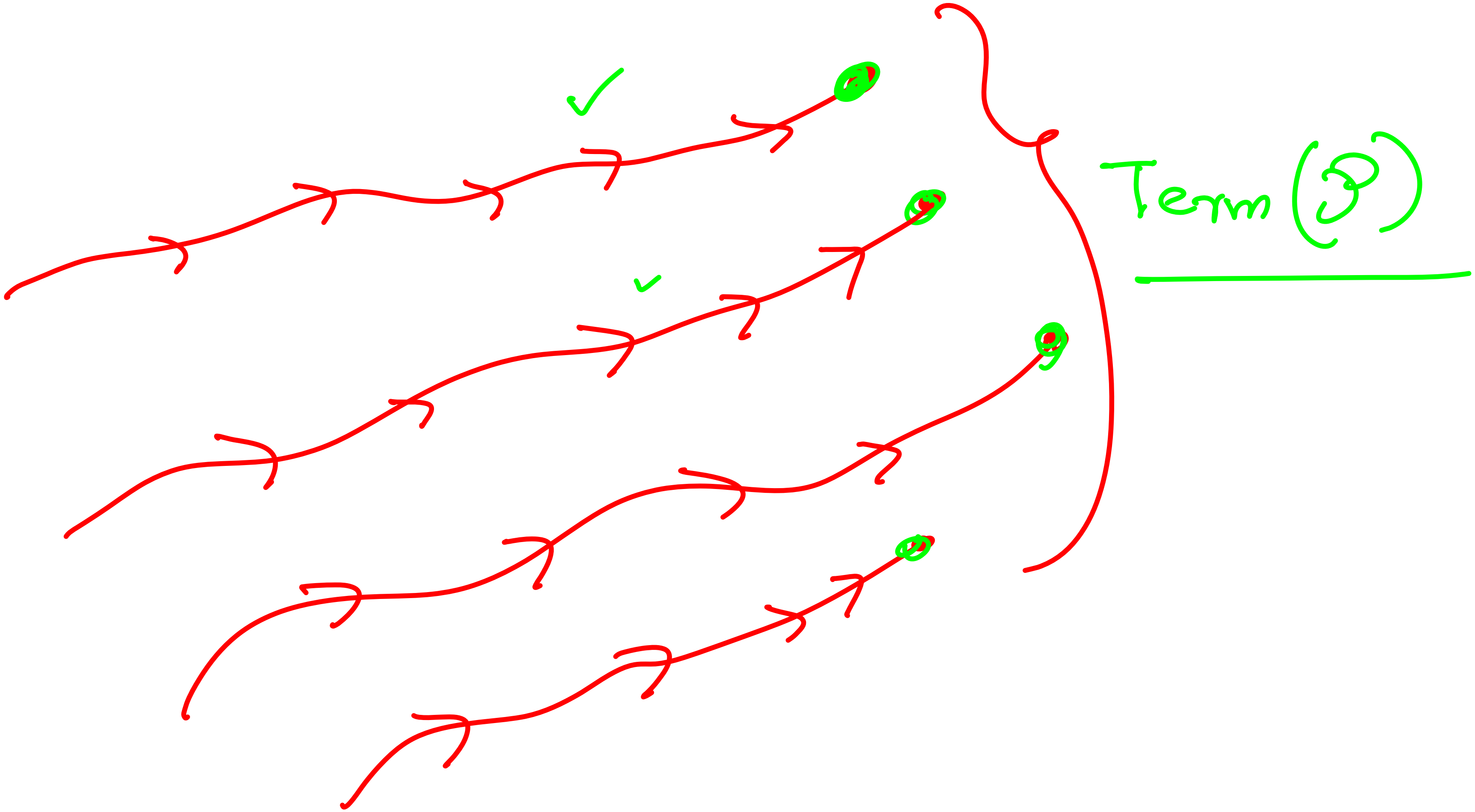
T is a maximal indepent set

$y \in V(G) - T$

$\cancel{T \cup \{y\}}$ ← not independence

$\mathcal{P}$ is path cover

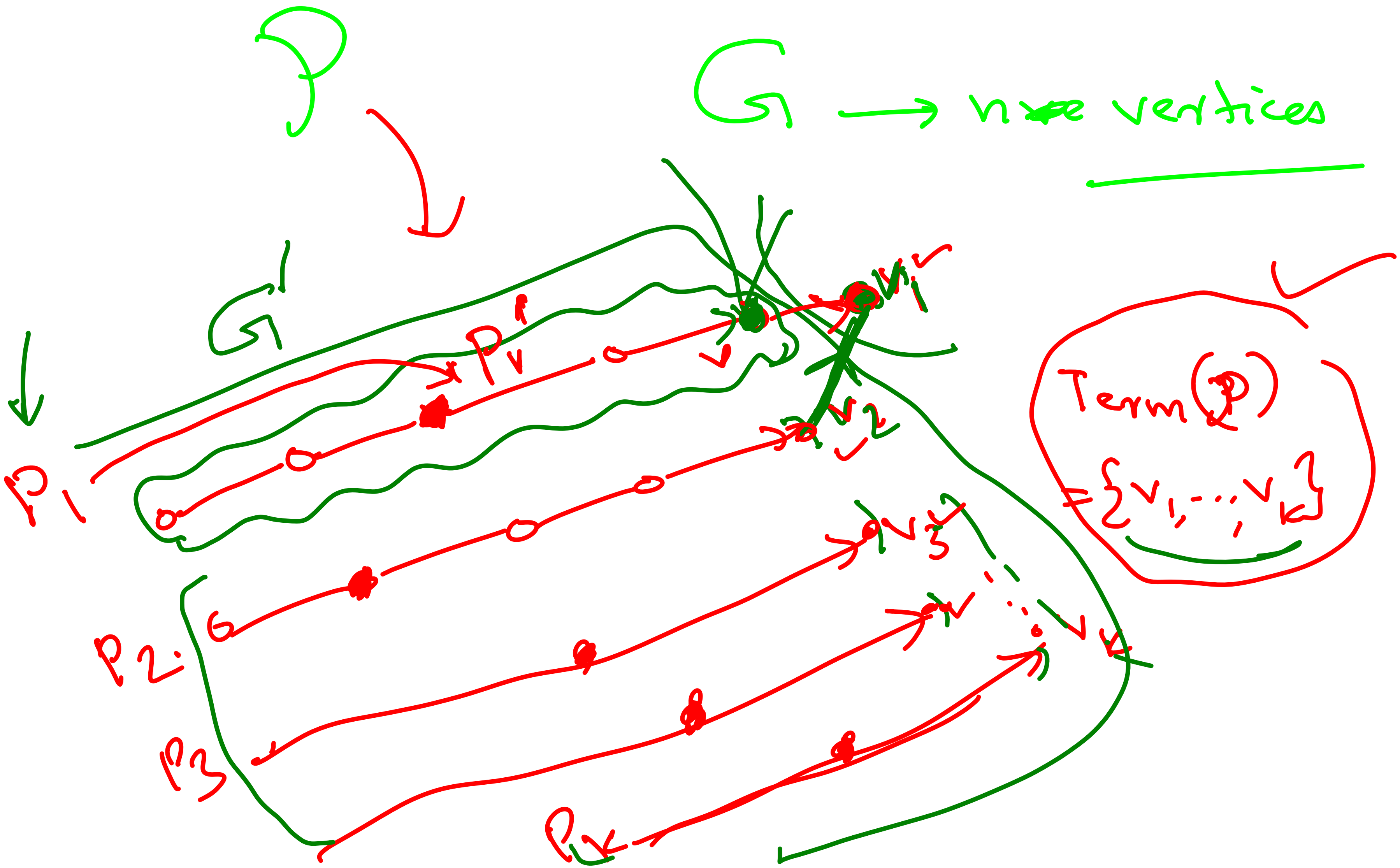
$p \in \mathcal{P}$, $\xcancel{\mathcal{P} - \{p\}}$

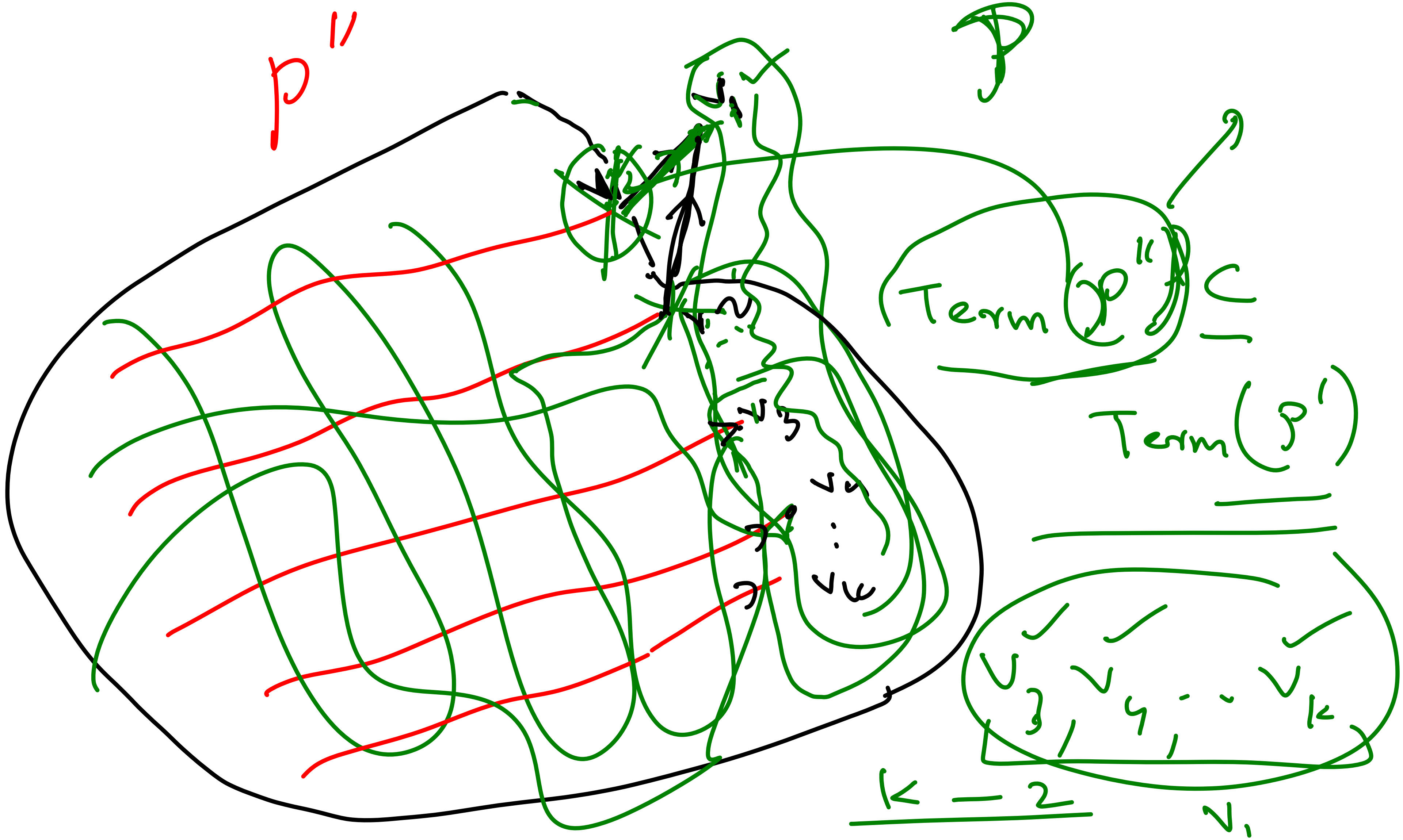

Term($\wp$)

P is a $\boxed{\text{minimal path cover}}$

if $\nexists$ no other P' such

that $\boxed{term(P') \subset term(P)}$


P
G → n vertices
G'
P_v'
P_1
P_2
P_3
P_k
v_1
v_2
v_3
v_k
Term(P)
$= \{v_1, \ldots, v_k\}$



P''
$\mathcal{P}$
v_1
v_3
v_4
v_k
$\mathrm{Term}(P'') \cup C$
$\mathrm{Term}(P')$
$v_3, v_4, \ldots, v_k$
$k-2$
v_1

a, b, c, d, e, f

$a \le b \le c \le d \le e \le f$ ✓

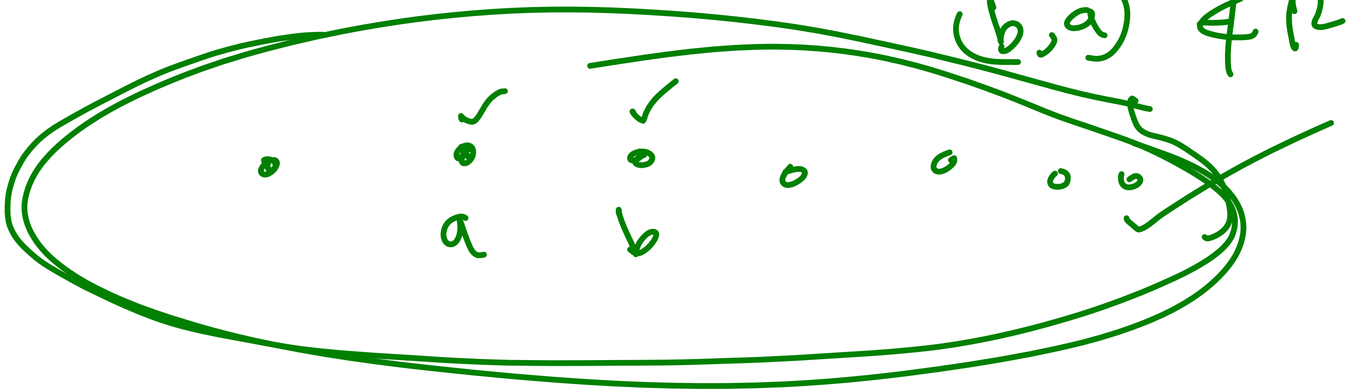
$(a,b) \in R$, $(b,c) \in R$

$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \ldots$

anti chain

$(a,b) \notin R$

$(b,a) \notin R$

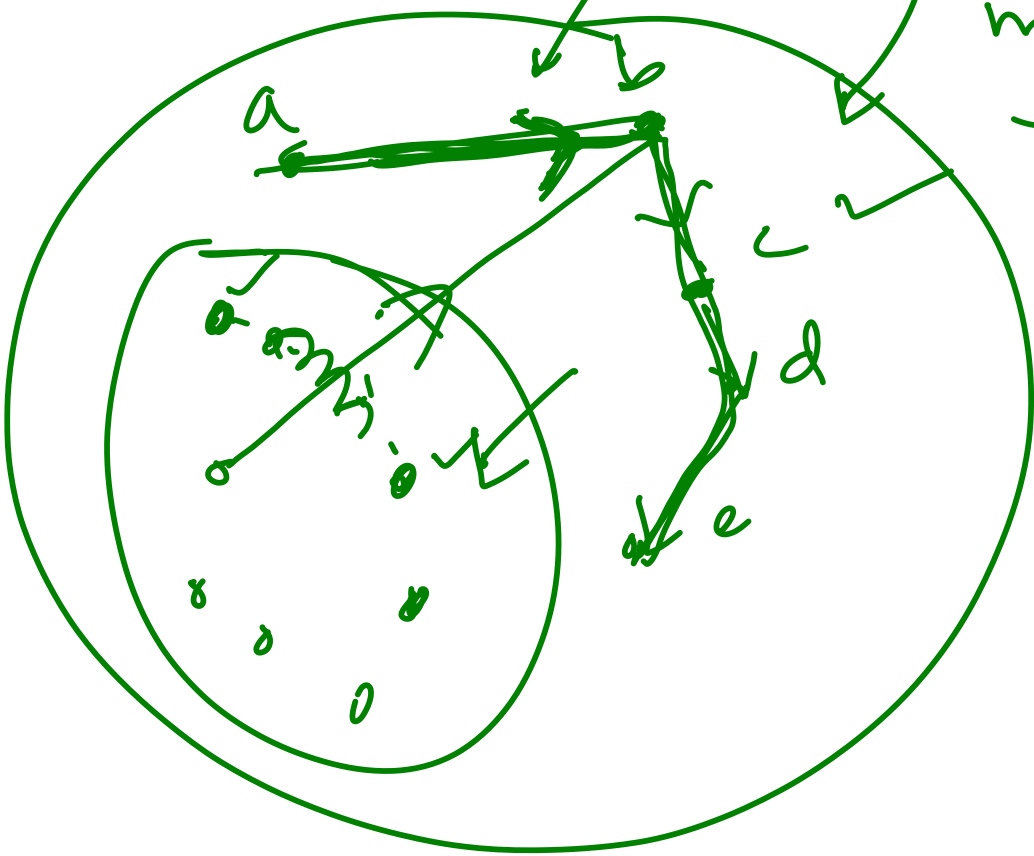


$(P, \leq)$

Dilworth's theorem

min # chains

= max cardinality # an antichain



$\alpha(G)$

the card. of the max antichain

$=$

$\geq$

$\leq$

MBPC(G)

min # chains to cover the partial order